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Spatial and Temporal Coherence of Sunlight

Physical intuition of spatial and temporal coherence:

Two waves are *temporally coherent* if they have the same phase difference and the same frequency. Two points in a waveform are said to be *spatially coherent* if they share a fixed-phase relationship. We begin this short essay by building the physical intuition behind the above two sentences, then move on discuss the temporal and spatial coherence of sunlight as seen to an observer on earth.

Figure 1 (left) shows a temporally coherent wave (Re{$e^{i\omega t-\phi}$}) with an infinite coherence time ($\tau_c = \infty$). The technical definition of coherence time varies depending on the system in question, and the system's parameters that can be experimentally measured; however, all definitions refer to the same concept, that is, the number of seconds over which the wave's true phase $\omega(t)*t - \phi(t)$ is suitably matched to a single frequency wave of best-fit $\omega_0*t - \phi$. In other words, the duration over which the wave can be considered to be monochromatic.

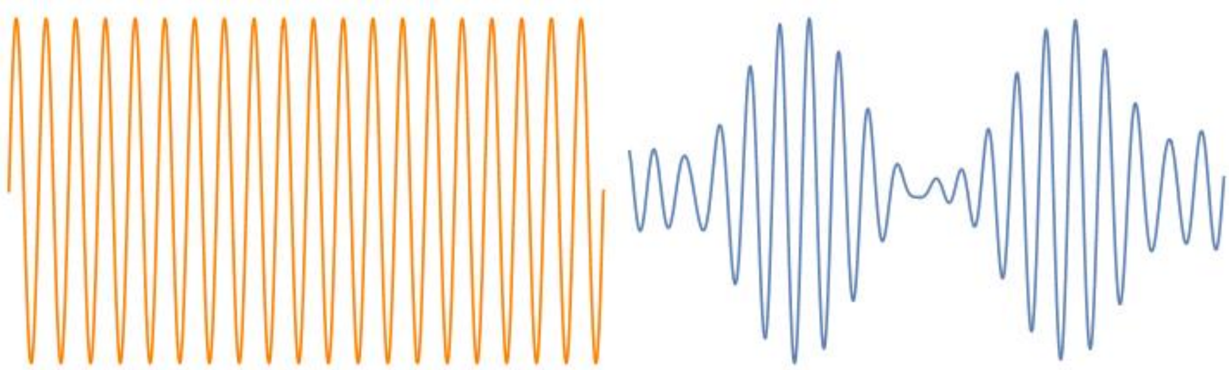


Figure 1: (Left) Monochromatic light source with infinite temporal coherence. (Right) Quasi-monochromatic light source with finite temporal coherence.

A wave with finite temporal coherence is shown in Figure 1 (right). By looking at the overlap of this non-monochromatic wave with a monochromatic wave of best fit (Figure 2), we see that the two only overlap for a finite period of time- this duration is τ_c, the coherence time. The most common way to measure τ_c is with a Michelson interferometer. When two waves overlap, interference (i.e. constructive or deconstructive addition) will only occur if the two waves share a fixed-phase relationship. In a Michelson interferometer, a wave is allowed to overlap with a time-delayed version of itself, and the interference of the two waves is observed on a screen. This time-delay is controlled, and therefore τ_c can be experimentally determined by observing the time-delay over which the interference pattern shows significant contrast.

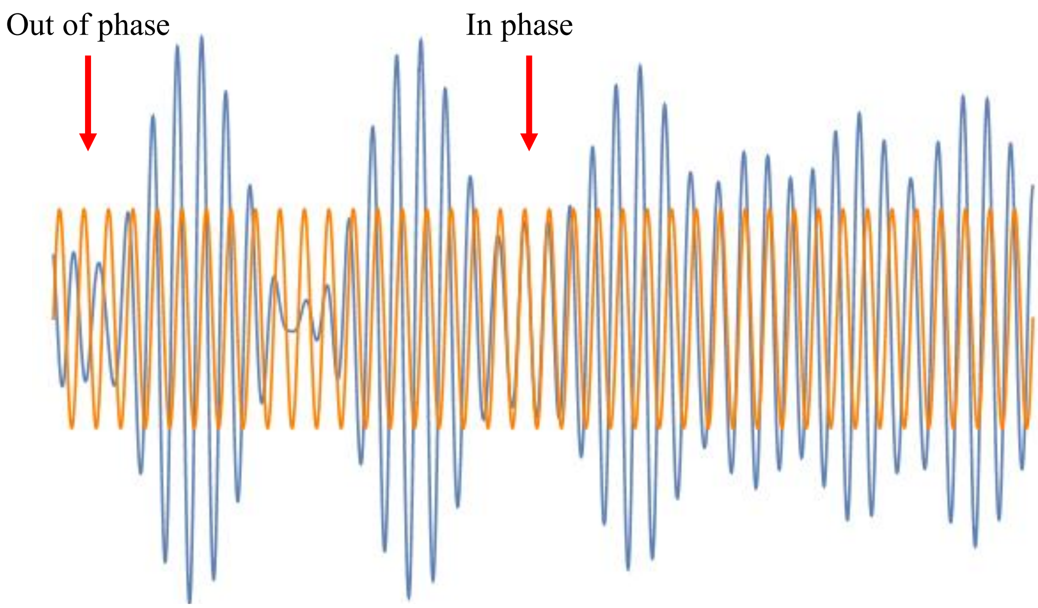


Figure 2: Overlap of monochromatic and quasi-monochromatic light sources. The coherence time is the duration over which the field looks approximately monochromatic

The spatial coherence of a wave also refers to a fixed-phase relation between two points on a wave, but instead comparing two points displaced in time, we compare two points displaced in space. To gain a physical intuition of spatial coherence, consider a non-monochromatic point source generating a light wave in the positive x direction, as shown in Figure 3. The lines in the figure signify points of constant phase, for example a peak or a trough in the wavefront. Even though the lines are not equally spaced (non mono-chromatic), notice there is still a fixed-phase relationship between P_1 and P_2; therefore, these points are considered to be spatially coherent.

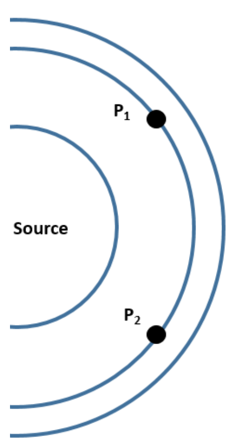


Figure 3: Wavefronts from non-monochromatic light source.

A Young's interferometer is typically the tool of choice to measure the spatial coherence of a light wave. In this set-up, two pinholes are separated by a distance *d* on a surface normal to the light source. The light wave passes through the two pinholes and the interference pattern is observed on a screen behind the pinholes. Since the two pinholes lay on a single plane normal to the light source, light must travel equal distance to arrive at each pinhole. Again, interference is only observed when the wave points share a fixed-phase relationship, so the degree of spatial coherence can be determined by the contrast of the interference pattern. It should be noted, to share a "fixed-phase relationship", it is not necessary for the points to have equal phase. Indeed, the points can be 90 degrees out of phase, and still show interference so long as the 90 degree separation does not vary significantly over time. In a Young's interferometer, *d* can be varied over a two-dimensional area:

$$A = \pi D^2$$

where *A*, the area over which interference contrast is significant, is called in the coherence area. *D* is the transverse coherence length.

For an example of spatially *incoherent* light, consider two radiating point sources. Notice if both sources are monochromatic with matched phase (as seen in Figure 4 left), then the points P_1 and P_2 have a fixed-phase relationship and are spatially coherent. The distance the top wave travels to P_1 is equal to the distance the bottom wave travels to P_2; and likewise for the points further away. Thus, these two points should have identical phase-relation for all of time— the points are spatially coherent. However, if one wave is non-monochromatic (bottom wave in Figure 4 right), the waves reach P_1 and P_2 at different times, and the spatial coherence is lost.

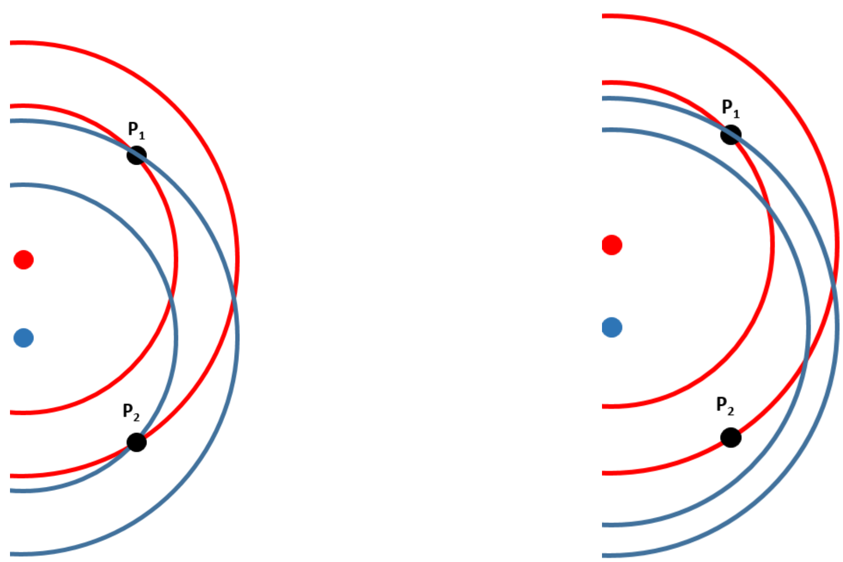


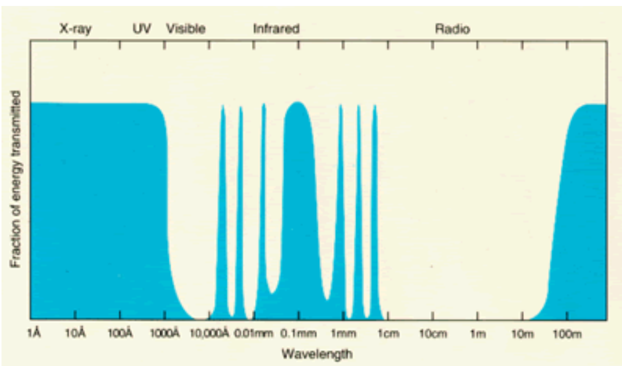
Figure 4: (Left) Wavefronts from two monochromatic point sources. (Right) Wavefronts from a monochromatic point source and non-monochromatic point source.

One can imagine a thermal light source as a composition of millions (or ~10^{56} in the case of the sun) tiny atoms that are radiating as point sources temporally independent from one another, so the light is certainly spatially incoherent in the near-feild. However, if the thermal source is far away (or small) then the spacing between P_1 and P_2, as determined by *d* in the

Young's interferometer, approaches a single point in relation to the thermal source, and the spatial coherence increases (as seen by the observer).

Spatial and temporal coherence of sunlight, as observed on earth:

The mechanism for light-emission within the sun is the acceleration of electrons and protons on the plasma near the sun's surface. Since the sun is a nearly-perfect blackbody with high absorption, the gamma rays produced in fusion at the sun's core are reabsorbed by outer layers, and the thermal motion of charge carriers at the surface plasma ultimately emit the light that propagates into free space and reaches the earth. Of the light that reaches the atmosphere, most UV and X-rays are absorbed by the atmosphere (Figure 5 left) to produce the solar spectrum as seen on the surface of earth (Figure 6 right):



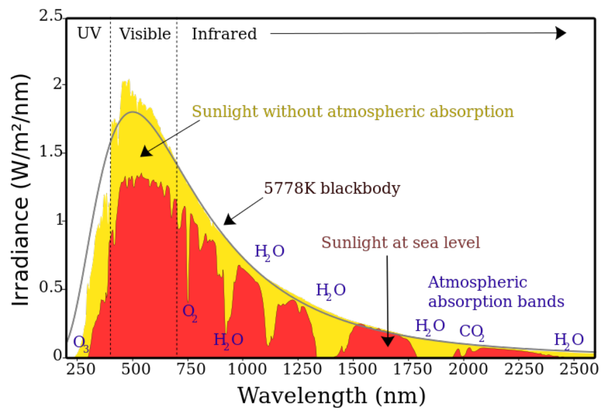


Figure 5: (Left) absorption profile of the earth's atmosphere, figure obtained from source [1]. (Right) the solar spectrum. Yellow is to an observer at the top of the atmosphere, red is to an observer on earth's surface, and the black line is the blackbody curve of a blackbody at the sun's surface temperature T = 5778 K, figure obtained from source [2].

There is no mechanism temporally synchronizing the motion of free electrons and ions on the sun's surface plasma, so the free charges emit independently from one another. Thus, sunlight is essentially temporally incoherent, indeed the coherence length- which is approximately the mirror displacement in a Michelson interferometer that causes fringe contrast to diminish- of white light is approximately 3λ [3-4].

In determining the spatial coherence of sunlight, there are two approaches one can take: apply a filter to make the light quasi monochromatic (referred to as the 'spectral degree of spatial coherence'), or directly measure the spatial coherence of the broadband source. The prior is usually measured by

. However, considering its proximity to earth, sunlight is surprisingly spatially coherent. As detailed in the experimental section, spatial coherence depends on two points in space sharing a fixed-phase relationship. This

The typical way of estimating if observer is the near-field or far-field of a light wave is with the dimensionless Fresnel number:

$$F = \frac{a^2}{\lambda\, r}$$

where a is the radius of the source, λ is the wave length of radiation, and r is the distance from the source to the observer. $F > 1$ is generally considered the near field, and $F < 1$ is the far-field where spatial coherence is lost. For visible light from the sun to an observer on earth is approximately 2×10^9, far greater than unity. In fact by the Fresnel number criterion, the earth-sun distance would need to be 3×10^{23} km for the earth to be in the sun's far-field, much further than the actual distance of 1.5×10^8 km. Despite this

Citations:

[1] The Columbus Optical SETI Observatory. http://www.coseti.org/photomap.htm

[2] Wikicommons https://commons.wikimedia.org/wiki/File:Solar_spectrum_ita.svg

[3] Hecht, E., 'Optics', 4th ed., 2002, Addison-Wesley

[4] Mikhailov, Anton. "The Michelson Interferometer: Determining Doublet Wavelengths, Coherence Lengths, and Refractive Indices of Gases." British Academic Written English (Physical Sciences), Apr. 2014.